YIELD SHEARING STRESS IN AN EMULSION

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A formula is derived for calculating the yield shearing stress in a thick emulsion.

When two immiscible liquids flow through a pipe, they form an emulsion as a result of turbulent mixing. An example is the emulsification of petroleum in wells during inundation with water.

Petroleum emulsions in water are usually stabilized by a surface-active substance dissolved in one of the liquids. Emulsions vary in stability, depending on the strength of the adsorption films on the surface of globules. It has been established in a study dealing with the rheological properties of stable emulsions [1] that they obey the Shvedov–Bingham law of viscous friction, which in the simplest case of plane laminar rectilinear flow along the 0x axis can be written as

$$\tau = \tau_0 + \mu \frac{du}{dn} \text{ when } \tau > \tau_0; \tag{1}$$

when $\tau < \tau_0$, the fluidity vanishes ($du/dn = 0$).

When the strength of adsorption films on the globule surfaces is low, which can occur, for example, during intratubular deemulsification following the removal of asphaltic–tarry substances from the globule surfaces by a deemulsifier, then the emulsion loses much of its stability and becomes stratified so that it is impossible to study its rheological properties under static conditions.

Depending on the concentration of the dispersed phase, emulsions are classified into thin and thick ones. In thick emulsions the globules are in tight contact with one another and, when such an emulsion is under shear, the globules are deformed and this requires an extra stress τ_0.

In order to determine the yield shearing stress τ_0, we will use the following model of a thick emulsion. A thick emulsion will be defined as one where the dispersed phase consists of spheres equal in diameter and packed so that the centers of every eight in a cluster form a rhombohedron. The various possible configurations of the spheres fluctuate between two extreme ones: the most closely packed and the most loosely packed (without disruption of the contact). The acute angle Θ of the rhombi formed by the rhombohedron edges varies from $\pi/2$ to $\pi/3$. The volume fraction of the dispersed phase β in such a thick emulsion can be calculated as follows [2]:

$$\beta = \frac{\pi}{6(1-\cos\Theta)\sqrt{1+2\cos\Theta}}. \tag{2}$$

As Θ varies from $\pi/2$ to $\pi/3$, according to (2), the volume fraction of the dispersed phase in a thick emulsion varies from 0.524 to 0.741. When its concentration falls below that, the globules will not be in tight contact with one another but can, instead, move freely relative to one another. As the concentration of the dispersed phase increases, we will consider that a phase reversal occurs and the dispersing phase becomes the dispersed one, with the concentration of the latter dropping below 0.259. Thus, according to our model, an emulsion is thick only as long as the concentration of its dispersed phase remains within 0.524-0.741.

We place a thick emulsion between two plane-parallel plates, one stationary and the other moving at a constant velocity. We refer the Cartesian coordinates to the stationary plate so that the latter lies in the x0y plane while the other plate moves along the 0x axis. We single out a volume element xyz of the

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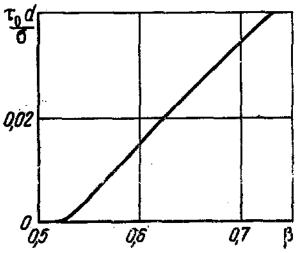

Fig. 1. Relation between $\tau_0\sigma/d$ and the concentration of the dispersed phase.

emulsion, in the shape of a rectangular parallelepiped bounded by planes parallel to the Cartesian planes at distances x and y, respectively, from them. The number of globules in this volume is

$$n = \frac{6\beta xyz}{\pi d^3}. \tag{3}$$

We will consider all globules to deform under shear, changing their shape from spherical to ellipsoidal. The degree of globule deformation depends on the magnitude of the acute rhombus angle and, conforming to the geometrical concepts introduced here, the minor semiaxis of the ellipsoid will be

$$b = \frac{d \sin\alpha}{2 \sin\Theta}, \tag{4}$$

where

$$\alpha = \arccos \frac{\cos\Theta}{\cos\frac{\Theta}{2}}. \tag{5}$$

The major semiaxis can be determined from the condition that the globule volume remains invariant:

$$a = \frac{d \sin^2\Theta}{2 \sin^2\alpha}. \tag{6}$$

Furthermore, using the well-known formulas for the surface area of a sphere and an ellipsoid [3], we find the amount of surface variation per globule as the difference between the surface area of a sphere and that of an ellipsoid:

$$\Delta f = \frac{\pi d^2}{2}\left[\frac{\sin^2\alpha}{\sin^2\Theta} + \frac{\arcsin\sqrt{1 - \frac{\sin^6\alpha}{\sin^6\Theta}}}{\frac{\sin\alpha}{\sin\Theta}\sqrt{1 - \frac{\sin^6\alpha}{\sin^6\Theta}}} - 2\right]. \tag{7}$$

The work expended on deforming all globules contained in the said volume element of emulsion is

$$A = \Delta f n \sigma. \tag{8}$$

The extra stress necessary for the initial shear is

$$\tau_0 = \frac{\Delta f n \sigma}{xyz \operatorname{ctg}\Theta}, \tag{9}$$

where xy denotes the area element on the moving plate to which an extra force is applied necessary for deforming the globules within the volume element, and where $z \cot\Theta$ denotes the displacement of the moving plate which will result in a complete deformation of these globules.

Inserting (3) and (7) into (9), and taking into account formula (2), we obtain

$$\tau_0 = \frac{\pi \operatorname{tg}\Theta}{2(1-\cos\Theta)\sqrt{1+2\cos\Theta}}\left[\frac{\sin^2\alpha}{\sin^2\Theta} - \frac{\arcsin\sqrt{1 - \frac{\sin^6\alpha}{\sin^6\Theta}}}{\frac{\sin\alpha}{\sin\Theta}\sqrt{1 - \frac{\sin^6\alpha}{\sin^6\Theta}}} - 2\right]\frac{\sigma}{d}. \tag{10}$$

The coefficient σ/d in (10) is a function of the concentration of the dispersed phase, as shown in Fig. 1. This curve can be approximated, within a 3% accuracy, by a straight line

$$\frac{\tau_0 d}{\sigma} = 0.195\beta - 0102. \tag{11}$$

We thus have an equation for calculating the yield shearing stress in a thick emulsion

$$\tau_0 = (0.195\beta - 0.102)\frac{\sigma}{d} \quad \text{for} \quad 0{,}524 < \beta < 0.741.$$

NOTATION

x, y	are the distances between planes bounding a volume element of emulsion;
z	is the distance between plane-parallel planes;
τ	is the shearing stress;
τ_0	is the yield shearing stress;
μ	is the dynamic viscosity;
du/dn	is the shear rate gradient;
Θ	is the acute angle of the rhombohedron sides formed by globule centers;
n	is the number of globules contained in the volume element;
a, b	are the ellipsoid semiaxes;
d	is the diameter of a globule;
Δf	is the change of globule surface due to deformation;
A	is the work expended on deforming all globules within the volume element of emulsion;
σ	is the interphase stress;
β	is the volume fraction of the dispersed phase.

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